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Uncertainties on Asteroid Albedos Determined by Thermal Modeling

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ABSTRACT

We present an analysis of the accuracy of geometric albedos determined for asteroids through the modeling of observed thermal infrared radiation. We show that albedo uncertainty is dominated by the uncertainty on the measured H_V absolute magnitude, and that any analysis using albedos in a statistical application will also be dominated by this source of uncertainty. For all but the small fraction of asteroids with a large amount of characterization data, improved knowledge of the H_V magnitude will be fundamentally limited by incomplete phase curve coverage, incomplete light curve knowledge, and the necessary conversion from the observed band to the V band. Switching the absolute magnitude standard to a different band such a r' would mitigate the uncertainty due to band conversion for many surveys, but this only represents a small component of the total uncertainty. Therefore, techniques making use of these albedos must ensure that their uncertainties are being properly accounted for.

1. Introduction

Thermal infrared sky surveys have produced infrared measurements of a large number of the known asteroids in the inner Solar system. Application of thermal models to these data have resulted in diameter and albedo constraints for over 100,000 asteroids from IRAS (Tedesco *et al.* 2004), AKARI (Usui *et al.* 2011), Spitzer (Trilling *et al.* 2010), and WISE/NEOWISE (Mainzer *et al.* 2011a) combined. Mainzer *et al.* (2015) present an overview of space-based studies of asteroids in the infrared, including a discussion of the techniques for thermal modeling.

Thermal infrared observations are primarily sensitive to the size of the asteroid observed. Once the orbit of the body is constrained, the thermal infrared flux is directly associated to the size via the thermal model used. Across a range of compositions and optical reflectivities, the emissivity of asteroids is consistently very close to a value of ~ 0.9 (Lim *et al.* 2005; Vernazza *et al.* 2012). As shown by (Harris & Harris 1997), this means that changes to the measured absolute magnitude of an asteroid have only a minor effect on the calculated diameter.

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Masiero *et al.* (2018) presented an analysis of the accuracy of infrared diameters using the distribution of albedos observed for asteroid families. They found that the uncertainty on the H_V absolute magnitude is a significant component of the overall albedo uncertainty, and dominates the albedo uncertainty for typical H_V accuracies. This means that the knowledge of the absolute magnitude plays a critical role in our understanding of albedos.

Pravec *et al.* (2012) performed an analysis of the H_V values published in asteroid orbital catalogs (the most common source for these values) compared to objects tracked over long periods of time with photometrically-calibrated systems. They showed that while H_V values found in orbit catalogs are generally good to a few-tenths of a magnitude for large and well-studied asteroids, smaller objects can show significant errors, both random and systematic, at the level of 0.5 mag. This is a result of a combination of effects, including using as assumed value for the phase curve G_V parameter (Bowell *et al.* 1989), the absolute photometric calibration of the surveys providing photometry, some surveys using unfiltered observations for photometry, the accuracy of the estimation of mean brightness (to account for rotational light curve effects), changing viewing aspects resulting in different views of the asteroid's 3D shape, and the accuracy of the conversion from the observed band to the 'standard' V band used for H_V calculation. For each of these uncertainty components, observations can reduce their individual contribution (such as was done by Pravec *et al.* 2012), however this requires large amounts of telescope time for each object to densely sample the rotational light curve, phase curve, and different apparitions to constrain the 3D shape. The majority of objects in Minor Planet Center's orbital catalog do not, and will not, have this level of knowledge without targeted densely-sampled followup covering a broad range of phase angles to constrain both the light curve and phase curve.

Here we investigate the effect that uncertainties on H_V will have on the albedos derived when a survey provides diameter fits, as occurs with thermal infrared data. This is important to help us better understand the limitations of the derived albedo data sets, and how these uncertainties will affect our interpretations of the population as a whole and the sub-populations within the asteroids.

2. Relationship between diameter, absolute magnitude, and albedo

The empirical relationship between the size of a body, its geometric albedo, and the brightness is often described (e.g. Harris & Lagerros 2002) as:

$$D_{km} = C_V \, \frac{10^{-H_V/5}}{\sqrt{p_V}} \tag{1}$$

which can be rearranged as:

$$p_V = C_V^2 \, \frac{10^{-H_V/2.5}}{D_{km}^2} \tag{2}$$

where D is the size of the body in kilometers, H_V is the phase- and distance-corrected magnitude (i.e. absolute magnitude) in the V band, and p_V is the geometric albedo in the V band. The constant parameter C_V is usually taken to be $C_V = 1329$ km (for example, see the derivation in Pravec & Harris 2007).

The definition of geometric albedo is the ratio of the true scattering of light by the surface compared to an ideal scatterer, here a disk of area $\pi r^2 = \pi \frac{D^2}{4}$ that is 1 AU from the sun, 1 AU from the observer, and at phase of $\alpha = 0°$. Following (Jewitt *et al.* 2013), this relationship can be written:

$$\pi \frac{D^2}{4} = (1.496e8\ km)^2 \frac{\pi}{p_V}\ 10^{0.4(V_\odot - H_V)}$$

$$D = 2.99e8\ km\ \frac{1}{\sqrt{p_V}}\ 10^{0.2V_\odot} 10^{-0.2H_V}$$

meaning the relationship constant C_V is:

$$C_V = 2.99e8 \times 10^{0.2V_\odot}\ km$$

The constant of interest is a function of the stellar apparent magnitude of the Sun in the band of interest, here $V_\odot$ band. Torres (2010) quote a value of $V_\odot = -26.76 \pm 0.03$, which they derive by recomputing the calibrations of Bessell *et al.* (1998) using updated reference stars. From this, we then derive a constant of $C_V = 1330 \pm 18$ km. This implies that an albedo derived from a measured diameter and an H_V magnitude will automatically have a $\sim 2.8\%$ relative uncertainty from the uncertainty on the Solar V apparent magnitude, even before accounting for errors on D and H_V. The previously-derived $C_V = 1329$ value, based on a $V_\odot = -26.762 \pm 0.017$ mag from Campins *et al.* (1985), is within measurement uncertainties of the value that is obtained with current Solar magnitude measurements.

An important point is that this constant is a function of the bandpass being used. The majority of current and planned sky surveys do not use the Bessell V filter, instead having moved to a filter set similar to that of the Sloan Digital Sky Survey (SDSS, Gunn *et al.* 1998; Smith *et al.* 2002). The conversion from the survey band to V will add an additional component of systematic uncertainty to any albedos determined. Further, this conversion will depend on the (unknown) composition of the object, as asteroids with different spectral curves have different colors, meaning that the systematic uncertainty will be different for different classes of object making comparisons between populations more difficult.

One option to reduce this conversion error is for the community to transition asteroid absolute magnitudes and albedos to a band that dominates the ongoing survey photometry. For example, the r' band is typically the most sensitive to asteroids for ground-based surveys given their intrinsic brightness combined with filter responsivities. Using $r'_\odot = -27.05 \pm 0.03$ (Vega mags, Willmer 2018) leads to a new constant value of $C_{r'} = 1164 \pm 16$. This would, or course, require determination of the $H_{r'}$ absolute magnitude and the $G_{r'}$ phase parameter for all asteroids being studied, as well as a conversion technique to compare new albedos to literature p_V values. However, as surveys such as the Legacy Survey of Space and Time at the Vera Rubin Observatory (LSST Science Collaboration *et al.* 2009) begin producing large quantities of asteroid photometry over many years, these measurements will become possible. Given that the majority of asteroid photometry over the next decade will be obtained in r' or a closely calibrated band, it is worth careful consideration by the community whether now is the time to switch standards.

In counterpoint, there are arguments against switching standards as well. Foremost is the extensive amount of literature currently using H_V and p_V, as well as the numerous diagnostics that exist based on these parameters. In addition, the V band covers the peak of the distribution of reflected light from an asteroid. That makes it a closer analog to the true bolometric albedo, which is an important value needed for thermophysical modeling. Instead of switching standards, a concerted effort to provide accurate $V - r'$ indices for all surveys for a range of asteroid compositions might alleviate some of the problems created by the current system. Any change, of course, would require extensive community discussion and IAU approval.

3. Absolute magnitude uncertainty

Following Eq 2, we can see that the error on albedo will be a combination of the errors on diameter and absolute magnitude. While diameter error can be independently assessed based on comparisons between different determination methods (e.g. infrared modeling, radar modeling, or occultation chord fits), the true uncertainty on H_V is more difficult to validate against an independent dataset.

Following Bowell *et al.* (1989), the absolute magnitude can be determined from fitting the phase-magnitude relationship of the asteroid using the equations:

$$H_V = V_{obs} + 2.5\log_{10}((1-G_V)\ \Phi_1 + G_V\ \Phi_2) - 5\log_{10}(R\Delta) \tag{3}$$
$$\Phi_1 = \exp\left(-3.33\tan^{0.63}(0.5\alpha)\right)$$
$$\Phi_2 = \exp\left(-1.87\tan^{1.22}(0.5\alpha)\right)$$

where α is the phase angle, R is the heliocentric distance, and Δ is the geocentric distance at the time of observation. This is the simplified functional form adopted by the IAU (Marsden 1985), though a more precise calculation is presented by (Bowell *et al.* 1989) in their equation A4.

Other photometric phase functions have been developed, such as the H-G_1-G_2 system and the H-G_{12} system (Muinonen *et al.* 2010), however as these either have more parameters (in the case of the H-G_1-G_2 system) or non-linear behavior (in the case of the H-G_{12} system) they require more data to accurately fit the phase curve and thus will have comparable or lower accuracy for sparse data sets.

We note that in well-defined cases with extensive data and multi-parameter fits, like those presented in Muinonen *et al.* (2010), the uncertainty on H_V can be of order 0.02 mag (1-σ). However as the authors of that paper note, a number of factors commonly encountered with photometric data can impair the determination of H_V including changing geometry between apparitions, incomplete rotational coverage at each phase angle, coverage of only a narrow range of phase angles, and imperfect conversion of photometry from the observed band to V.

The work of Vereš *et al.* (2015) provides an ideal example of real world results from fitting absolute magnitudes to a large, photometrically-calibrated survey that is sparse in time. In that work, the vast majority of observations had individual photometric uncertainties < 0.1 mag, meaning that the individual observations did not place a fundamental limit on the accuracy of the H_V determination. Through Monte Carlo simulations of different rotation states, those authors found that their statistical uncertainty on H_V was 0.3 mag for objects with $H < 18$ mag (sizes larger than approximately 1 km) using the H-G relation from Bowell *et al.* (1989), or a slightly improved uncertainty of 0.25 mag under the H-G_{12} system developed by (Muinonen *et al.* 2010). For cases where diameters are measured with an infrared survey with a nominal accuracy of 10%, the above uncertainty on H_V would result in a relative uncertainty on albedo of $32-36\%$. In the converse case of an object with only optical observations and using an assumed albedo with perfect accuracy, this H_V uncertainty alone would propagate to an uncertainty on diameter of $13-15\%$, with additional non-random uncertainty from the accuracy of the assumed albedo used.

Vereš *et al.* (2015) note that the fits to the G_V slope parameter are significantly worse than the H_V fits in their work. The uncertainty on G_V depends on the span of phase angles covered, with coverages $> 20°$ showing significantly smaller uncertainties than those with coverage $< 10°$. In addition, objects that are only seen at high phase angles will have significant errors on H_V due to the large lever-arm that the fitted value of G_V has, as is often the case for near-Earth asteroids. Due to this uncertainty on G_V, H_V errors of ~ 1 mag can be expected for objects on Earth-like orbits. This level of uncertainty for the H_V value would correspond to an albedo uncertainty of $\sim 70\%$ when using a infrared-determined diameter. In the case of a diameter calculated from H_V and an assumed albedo, the uncertainty on this diameter will be $\sim 42\%$. As G_V is only weakly correlated with taxonomy (see Table 6 from Vereš *et al.* 2015), asteroid color measurements cannot dramatically improve this.

An example of this situation is shown in Fig 1. Here, we assume an asteroid is detected with $V = 21$ mag (with negligible measurement uncertainty) at a phase of $\alpha = 60°$, 1 AU from the Earth and Sun. Based on (Vereš *et al.* 2015) we drew random G_V parameters from a normal distribution following $G_V = 0.2 \pm 0.2$. This results in a median absolute magnitude of $19.00^{+0.29}_{-0.54}$ (uncertainties shown are the 84^{th} and 16^{th} percentiles). For objects typically observed at smaller phase angles, like Main Belt asteroids, this situation will not occur and H_V values will be better constrained. However, near-Earth objects are often observed at large phase angles, where this problem can result in significant uncertainties in their absolute magnitudes. For example, cadence simulations for the Vera C. Rubin Observatory's Legacy Survey of Space and Time (LSST) indicate that the twilight NEO survey, if carried out, would have approximately half of the survey fields at Solar elongations below 110°, resulting in NEO detections predominantly at large phase angles (Jones *et al.* 2020).

As the range of phase angles covered increases, the constraint on the G_V value for an object will improve, and thus the H_V determination will also improve, however this is a strong function of the phase angles at which the object is observed. A Monte Carlo simulation of four V-band observations (assuming zero photometric error and perfect accuracy on the assumed albedo) in the range $10^\circ < \alpha < 30^\circ$ yields a final uncertainty on H_V of 0.23 mag and minimum calculated diameter uncertainty of 11%, while four observations drawn from $50^\circ < \alpha < 70^\circ$ result in an uncertainty of 0.68 mag on H_V and 30% on D. Increasing the number of observations to 64 further improves the H_V uncertainty to 0.08 mag (4% on D) for $10 - 30^\circ$ and 0.31 mag (14% on D) for $50 - 70^\circ$. Photometric measurement uncertainty will increase the true uncertainty on both H_V and D, while the uncertainty on the assumed albedo will increase diameter uncertainty as well.

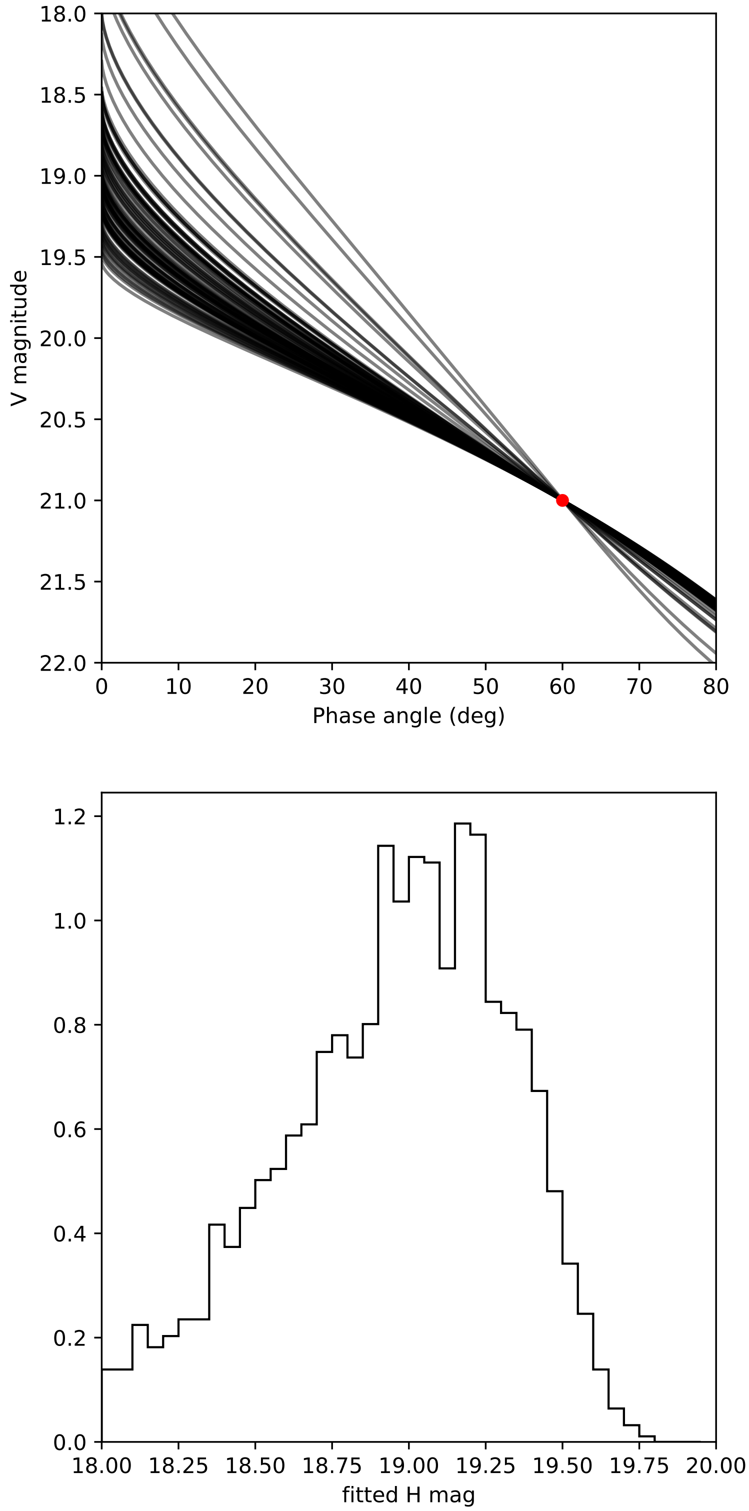

Fig. 1.— An example of the range of allowable phase curve fits for an asteroid detected only at high phase angles. (Top) The red circle shows the simulated detection, with possible H-G phase relations shown as gray lines. (Bottom) Normalized histogram of possible H_V values in this example for 2000 Monte Carlo simulations of the G_V parameter.

It should be noted that these results depend on the assumption that the H-G or H-G_{12} phase functions can adequately describe the opposition effect of all objects. Recent work by Mahlke *et al.* (2020) investigated the photometric phase curves of over 90,000 asteroids in two visible light bandpasses from the ATLAS survey. They show that even for objects with well-sampled phase curves, the difference in fitted absolute magnitude between the H-G_{12} system and the H-G_1-G_2 has systematic offsets of order 0.1 mag and random uncertainties of ~ 0.2 mag. This will directly impact the accuracy of the albedos determined for asteroids from thermal modeling.

4. Discussion

The overall uncertainty on any given asteroid's albedo measurement is a combination of the uncertainties on the H_V value and the diameter derived from thermal modeling. Comparisons of diameters determined from thermal modeling to those from different surveys as well as independent sources such as occultations and radar observations have shown that when multiple infrared measurements are available that sample the thermal emission portion of an asteroid's spectral energy distribution, the diameter uncertainty is approximately 10% (Mainzer *et al.* 2011b; Usui *et al.* 2014; Wright *et al.* 2018; Herald *et al.* 2020). This uncertainty is primarily caused by the deviation of the applied thermal model (usually a sphere with a simple temperature profile) from the asteroid's actual thermophysical properties. This propagates to an albedo uncertainty of $\sim 20\%$, which is comparable to the albedo uncertainty resulting from a typical H_V uncertainty. Along with the uncertainty on the constant in Eq 2 the result is a top-level albedo uncertainty of $\sim 28\%$ for typical values of well-studied objects of $\sigma_D = 10\%$ and $\sigma_{H_V} = 0.2$ mag.

Using large samples of objects with assumed uniform properties, e.g. from asteroid families (Masiero *et al.* 2015) or selected by photometric colors (Ivezić & Ivezić 2020), it is possible to obtain a mean albedo for the population that is known to higher precision than any single object's albedo. In this case, the uncertainty on a diameter inferred using this assumed albedo would be dominated by the accuracy with which the object has been assigned an albedo and the uncertainty on H_V. For a case of $\sigma_{H_V} = 0.2$ mag and an albedo assumed to be known with arbitrary precision, the uncertainty on diameter will be $\sim 10\%$. As shown by Pravec *et al.* (2012), ~ 0.2 *mag* is the smallest H_V uncertainty that can be reasonably expected from current data for objects with $D < 10$ km, even after correcting for systematic errors in the orbital catalogs. For objects that are newly discovered, light curve properties and phase curve behavior will not be characterized to high precision without many years of observations, and the characterization accuracy will depend on the the observing cadence. These objects will thus have commensurately worse H_V constraints, which would translate to larger uncertainties on inferred diameter even under the assumption of perfect albedo assignment. As discussed above, for newly discovered NEOs the diameter uncertainty from optical data alone can reach $\sim 50\%$, particularly for objects on Earth-like orbits.

5. Conclusions

As is noted by both Bowell *et al.* (1989) and Muinonen *et al.* (2010), the geometric albedo determined from the relation in Eq 1 might be more appropriately called a 'pseudo-albedo' as it is not a direct measurement, but rather inferred from models of models of measurements. This is not to say that it is not useful for population analysis or investigations of individual objects, as this is clearly demonstrated in the literature. Rather, as highlighted by Bowell *et al.* (1989), this relationship should be treated with caution as multiple assumptions go into a single derived value. As we have discussed here, the uncertainty on H_V can have a large impact on our knowledge of p_V, and it is nearly impossible to independently verify H_V measurements against other, non-photometric data sources. In light of this, we urge caution in attempting to derive physical properties from albedos alone.

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